The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: December 19, 2018 Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last three hours.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please turn off all cell phones.
- No headphones are allowed.
- All work should be performed on the exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	Point Value	Your score	Topic
1	14		Heart and Soul of Continuous Time
2	12		Discrete-Time System Identification
3	12		Continuous-Time Convolution
4	12		Discrete-Time Feedback System
5	12		Continuous-Time Circuit Analysis
6	12		Bluetooth Receiver
7	12		Continuous-Time Equalization
8	14		Discrete-Time Audio Effects
Total	100		

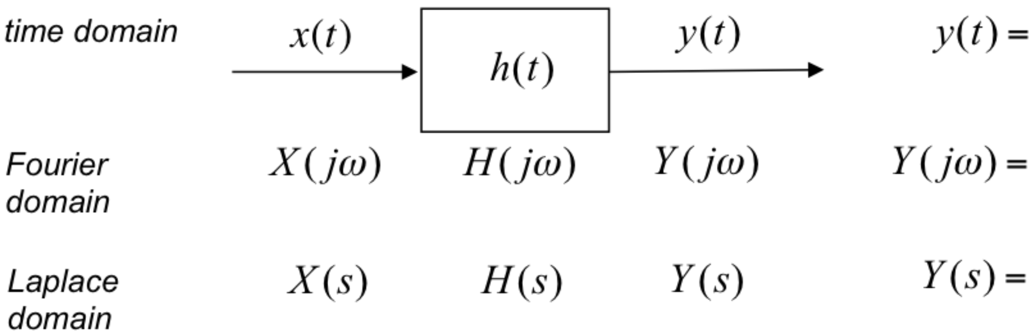
Problem 1. Heart and Soul for Continuous-Time Signals and Linear Systems. *14 points*.

(a) **LTI Systems**. Consider a continuous-time linear time-invariant system with input signal x(t), impulse response $h(t)$ and output signal $y(t)$. 9 *points*.

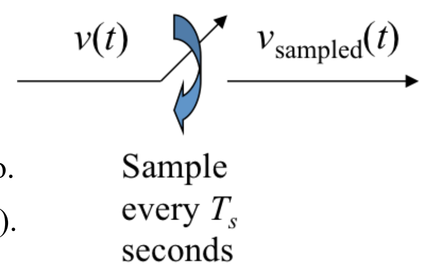
i. Give the relationship for $y(t)$ to $x(t)$ and $h(t)$ involving only operations in the time domain.

ii. Give the relationship for $Y(j\omega)$ to $X(j\omega)$ and $H(j\omega)$ using only operations in the Fourier (frequency) domain.

iii. Give the relationship for $Y(s)$ to $X(s)$ and $H(s)$ using only operations in the Laplace domain.



(b) **Sampling & Aliasing.** *5 points.*

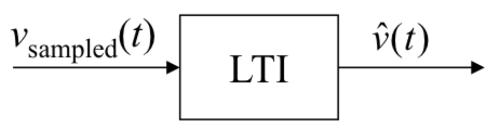


i. *Sampling*. Consider sampling modeled as an instantaneous closing and opening of a switch every T_s seconds.

When the sampling switch is open, assume $v_{\text{sampled}}(t)$ is zero.

Give a time-domain expression for $v_{\text{sampled}}(t)$ in terms of $v(t)$.

ii. *Reconstruction*. Describe the LTI system needed to extract an estimate of $v(t)$ from $v_{\text{sampled}}(t)$.



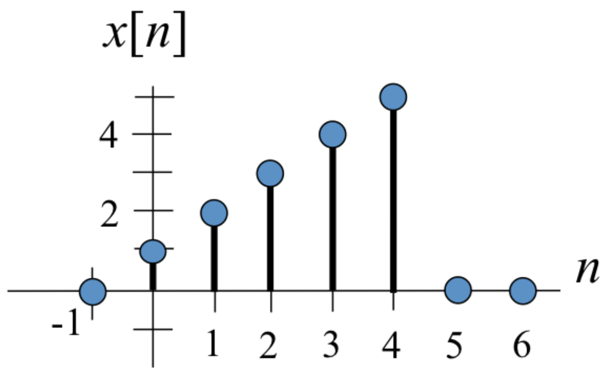
The estimate of $v(t)$ is denoted as $\hat{v}(t)$.

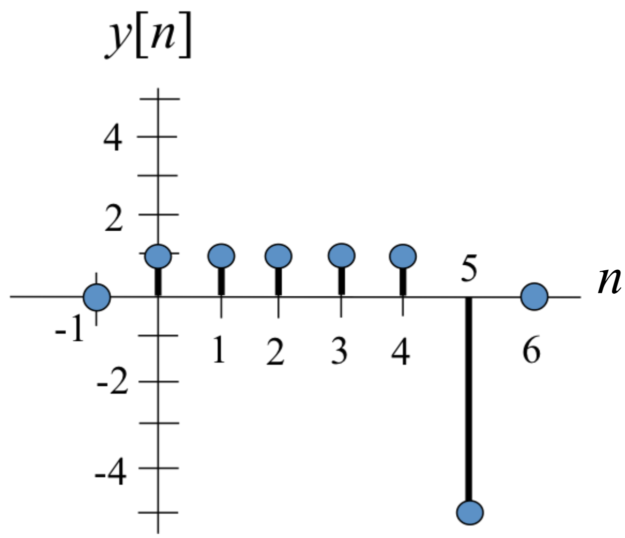
Over what frequencies is the estimate of $v(t)$ accurate?

Problem 2. Discrete-Time System Identification. *12 points*.

You are given a causal discrete-time linear time-invariant (LTI) system with unknown impulse response $h[n]$ to analyze.

When the five-sample causal signal $x[n]$ given below is input into the unknown system, the response $y[n]$ is six samples long and causal, as shown below.





x = [1 2 3 4 5 0 0];

y = [1 1 1 1 1 -5 0];

(a) Find $h[n]$. *9 points.*

(b) Plot $h[n]$. *3 points*.

Problem 3. Continuous-Time Convolution. *12 points*.

Convolve the two-sided continuous-time signals

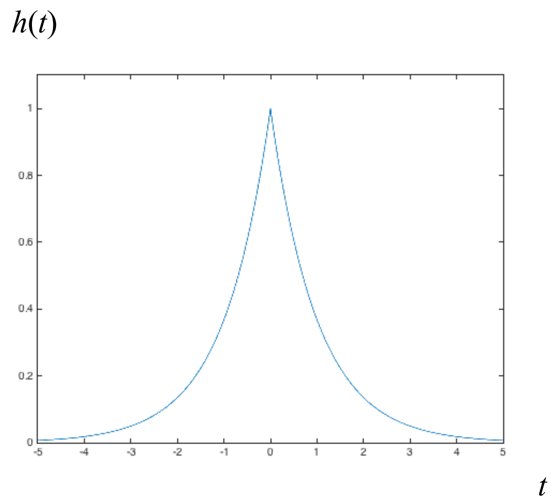
$x(t) = \cos(\omega_0 t)$ and $h(t) = e^{-a\,|t|}$

where a is real-valued and $a > 0$.

Both signals are defined for $-\infty < t < \infty$.

A plot of $h(t)$ over a finite interval of time is shown on the right for $a = 1$.

Please solve this problem for a general positive real value for a.



Problem 4. Discrete-Time Feedback System. *12 points.*

Consider a discrete-time linear time-invariant (LTI) system with input signal $x[n]$ and output signal $y[n]$ that is governed by the following second-order difference equation for $n \geq 0$:

$$y[n] = 1.8\, y[n-1] - K\, y[n-2] + x[n]$$

where K is a real-valued constant.

(a) What are the initial conditions of the system and what values should they have? *3 points*.

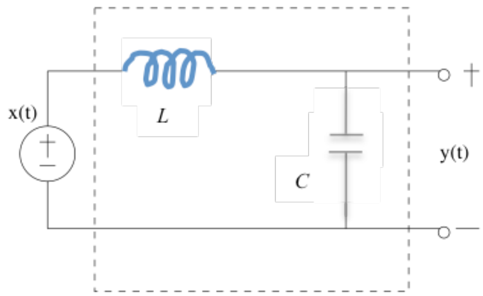
(b) Derive the transfer function $H(z)$ for the system, which will depend on K. *3 points*.

(c) Give the range of values for K for which the system is bounded-input bounded-output (BIBO) stable. *3 points*.

(d) Describe the possible frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch) that the system could exhibit for different values of K for which the system is BIBO stable. *3 points*.

Problem 5. Continuous-Time Circuit Analysis. *12 points*.

Consider the following analog continuous-time circuit with input voltage $x(t)$ and output voltage $y(t)$:



The initial voltage across the capacitor is 0 V and the initial current in the inductor is 0 A; hence, the circuit is a linear time-invariant system.

(a) Using the voltage drop around the loop

$$x(t) - L\frac{d}{dt}i(t) - \frac{1}{C}\int_{0^-}^{t} i(t)dt = 0$$

take the Laplace transform of both sides of the equation to find the relationship between $X(s)$ and $I(s)$. $I(s)$ is the Laplace transform of the current $i(t)$. *3 points*.

(b) Using the formula for the voltage across the capacitor

$$y(t) = \frac{1}{C}\int_{0^-}^{t} i(t)dt$$

take the Laplace transform of both sides and substitute the expression for $I(s)$ obtained in part (a) to obtain the transfer function $H(s)$ in the Laplace domain so that $H(s) = Y(s) / X(s)$. *3 points*.

(c) What is the impulse response $h(t)$? *3 points*.

(d) Is the system bounded-input bounded-output stable? *3 points*.

Problem 6. Bluetooth Receiver. *12 points.*

Bluetooth operates in the 2400-2499 MHz unlicensed frequency band.

At any given time, Bluetooth will transmit on one of 79 channels, and each channel is 1 MHz wide.

Channel k begins at $(2402 + k)$ MHz where $k = 0, 1, \ldots, 78$.

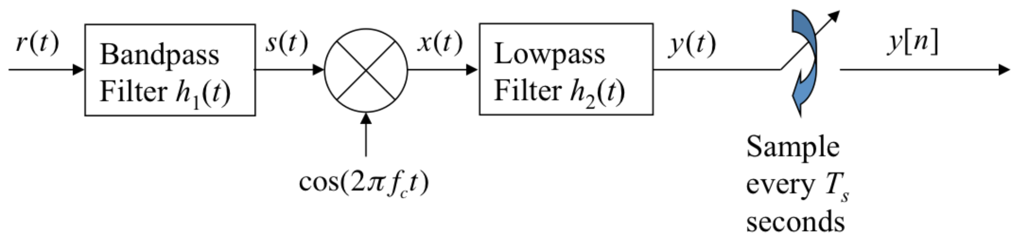
Bluetooth changes the 1 MHz channel on which it operates 1600 times/second to avoid interference.

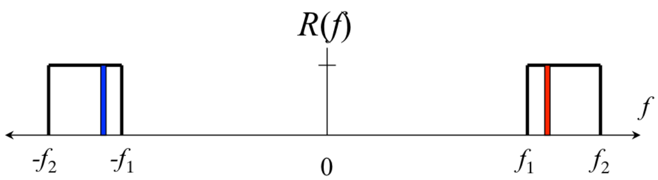
A Bluetooth receiver has two subsystems in cascade. The first subsystem involves continuous-time signal processing blocks and the second subsystem involves discrete-time signal processing blocks.

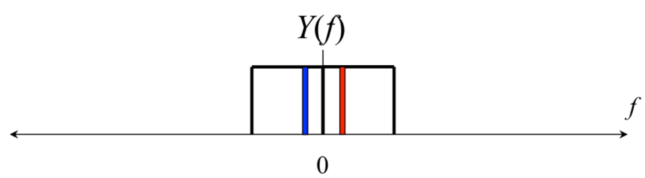
(a) **The continuous-time signal processing** blocks are given below, where $r(t)$ is the received radio frequency signal. In the plot for $R(f)$, one of the 1 MHz channels is shaded, and its counterpart in negative frequencies is also shaded. Demodulation produces $y(t)$, whose spectrum $Y(f)$ is below.

Let $f_1 = 2400$ MHz and $f_2 = 2499$ MHz.

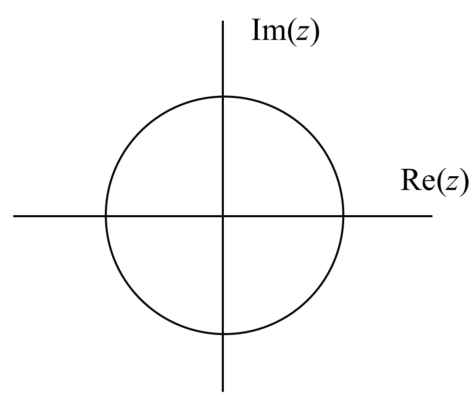
What is the demodulating frequency f_c? *3 points*.







(b) The first **discrete-time signal processing block** is filtering. Design a **second-order** linear time-invariant (LTI) infinite impulse response (IIR) filter to extract channel k from $y[n]$. Assume the sampling rate in part (a) is $f_s = 200$ MHz. Give formulas for, and plot, the two poles and two zeros. *9 points*.



Problem 7. Continuous-Time Equalization. *12 points.*

When sound waves propagate through air, or when electromagnetic waves propagate through air, the waves are absorbed, reflected and scattered by objects in the environment.

In the transmission of sound waves over the air in a room from an audio speaker to a microphone, we will model the direct path from the speaker to the microphone as having zero delay, and a one-bounce path from the speaker to an object and then to the microphone having delay t_1.

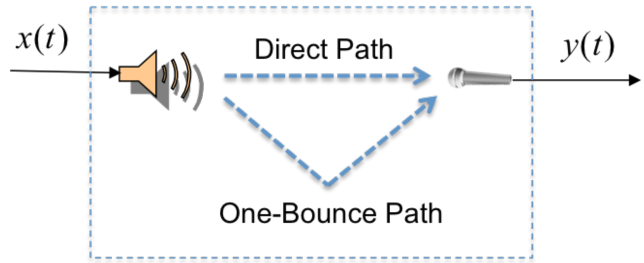
This single reflection is a type of echo.

We model the signal $y(t)$ at the output of the microphone as

$$y(t) = x(t) - \alpha\, x(t - t_1)$$

where α is a real-valued constant and $t_1 > 0$.

We model that system that connects $x(t)$ and $y(t)$ as linear and time-invariant (LTI).

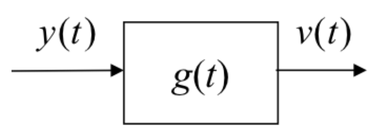


(a) Derive a formula for the impulse response $h(t)$. *3 points.*

(b) Find transfer function in the Laplace domain $H(s)$. *3 points.*

(c) We add an LTI filter at the microphone output to remove as much of the echo as possible. Design the continuous-time filter by giving its transfer function $G(s)$ in the Laplace domain. The filter must be bounded-input bounded-output (BIBO) stable. *6 points.*

a. Case I. $\alpha < 0$.

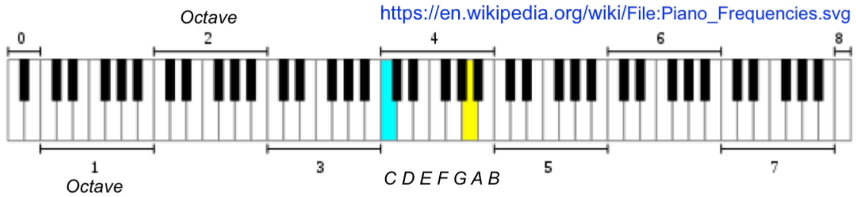


b. Case II. $\alpha = 0$.

c. Case III: $\alpha > 0$.

Problem 8. Discrete-Time Audio Effects. *14 points.*

The notes on the Western scale on an 88-key piano keyboard grouped into octaves follow:

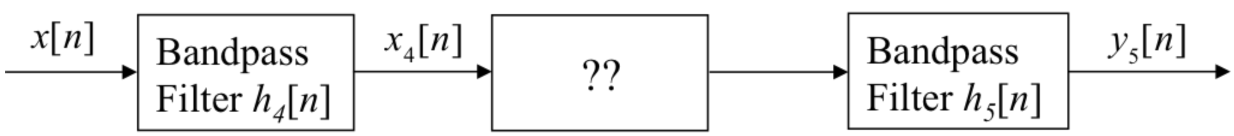


The frequency of note C6 (i.e. 'C' in the 6th octave) at 1046.5 Hz is twice the frequency of C5 at 523.25 Hz, and the frequency of C5 is twice the frequency of C4 at 261.625 Hz, and so forth.

This type of octave spacing occurs for all of the notes on the Western scale.

You are asked to design a **discrete-time** audio effects system that will extract each octave of frequencies and then alter that octave of frequencies to be in the next higher octave. All notes on the Western scale in the extracted octave should appear as the same notes in the next higher octave.

Here are the processing steps to extract the fourth octave in $x[n]$ and alter those frequencies to be in the fifth octave. Filter $h_k[n]$ represents the impulse response of the bandpass filter to extract the kth octave.



(a) For the bandpass filters, give an advantage of using finite impulse response (FIR) filters vs. infinite impulse response (IIR) filters. *3 points*.

(b) For the bandpass filters, give an advantage of using IIR filters vs. FIR filters. *3 points*.

(c) What operation (or operations) would you choose for the ?? block. How does your choice guarantee that any note on the Western scale in the fourth octave in $x_4[n]$ will appear as the same note one octave higher in $y_5[n]$? What additional audio effects would your choice create? *8 points*.